

The Xmath Calculator

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Xmath [1] and dMath [2] are projects supported by the European Commission through the Minerva Action (Xmath) [3] and the Leonardo da Vinci program (dMath) [4]. These programs seek to promote European cooperation in the fields of open and distance learning (Xmath) and vocational training (dMath). A pedagogic calculator [5] is being developed using *webMathematica* [6, 7] in connection with these programs. This calculator gives intermediate steps for calculations in a wide range of applications (integration, differentiation, algebra, equation solving, and so on). An expression is broken down and analyzed by *Mathematica* packages that have been developed to work with *webMathematica*. Rules that are familiar from hand calculation are given and so are the intermediate results. The user may scroll to look at a number of levels (the first level is the first hint) and then continue by hand. In the same way as an expression may be broken down into different levels, so can the calculation steps. This gives structured output, as a professor would do it on a blackboard, stating the rules at each level. The output in *webMathematica* may be written as MathMLForm, giving a non-image format that can be rendered in Internet Explorer using proper style sheets. One of the main ideas of Xmath is implementing MathML [8] in general web pages containing mathematical expressions. The dMath project creates a database of mathematical modules using the SciLas system developed in the project. The Xmath calculator is connected to the database modules and will be further developed in dMath.

■ **The Xmath Project**

The Xmath project is part of the Minerva Action. The Minerva Action's purpose is to increase European cooperation in information and communication technology (ICT) and open and distance learning (ODL) in education. The action seeks to promote understanding of the implications of ICT and the critical and responsible use of ICT for educational purposes among teachers, learners, decision makers, and the public at large.

At European universities and colleges the numbers of students in mathematical courses and passing the exams are dangerously low. To meet this challenge, mathematics teaching needs to undergo an innovative process, making new technology important. This will create more efficient, direct teaching and allow an explorative and personal way of working [9, 10].

Xmath has built a framework making it possible both to use MathML in mathematical education on the internet and to evaluate the pedagogical advantages of

net-based education in mathematics. Development and research into MathML's use in e-learning applications has been one of the main purposes of Xmath. Using MathML in Internet Explorer requires special style sheets and plug-ins (e.g., MathPlayer). MathML is a standard supported by the World Wide Web Consortium (W3C) [8]. They have supported the Xmath project as well. *webMathematica* [6] and *Mathematica* [11] are important parts of this framework.

A pilot course [1] has been developed to test and demonstrate the main ideas of Xmath. The Xmath project has been completed and reports on the pedagogical aspects of ODL in Xmath [12] and the Xmath project [13] have been written. The Xmath project arranged The First European Workshop on MathML & Scientific e-Contents [14]. The second workshop was arranged by the follow-up project, dMath [15].

■ The dMath Project

The dMath project is a follow-up to Xmath and is intended to be an industrialization of the Xmath project. This project will develop a European database of mathematical e-learning modules. These modules are independent but may be combined in an arbitrary way to set up a course in web-based mathematics.

The system contains three main parts:

1. The authoring suite (working title SciLas) has two pieces: an environment to write and store mathematical content trees, which are a collection of reusable learning objects organized in modules, and an environment to publish and download these learning objects with the capability to reorganize them into a course. The content trees are accessible through a web browser using predefined style sheets. Storage requires preliminary authorization and approval (dMath development server).
2. The dMathArchive, based on this system, is a collection of modules (see Figure 1).
3. The Xmath calculator, based on *webMathematica*, is to be used by developers, publishers, and course students to do calculations on the web. The calculator gives rules and steps in a calculation as a professor would do it on the blackboard.

The module design is important for adoption of local conditions and to enhance flexibility. Publishers may download modules from the archive in an arbitrary way and link them into a course in web-based mathematics. This collection of modules may be used as a mathematical dictionary as well.

In the dMath project we will focus on the needs for upgrading and furthering the education of teachers and engineers in mathematics. This is in accordance with national and European strategies for these target groups especially and for life-long learning in general. Because dMath is strongly innovative and uses new technologies (MathML, *webMathematica*, and innovative database design) other

potential users may be publishers, people in computer science, content providers of scientific e-learning material, and researchers and developers.

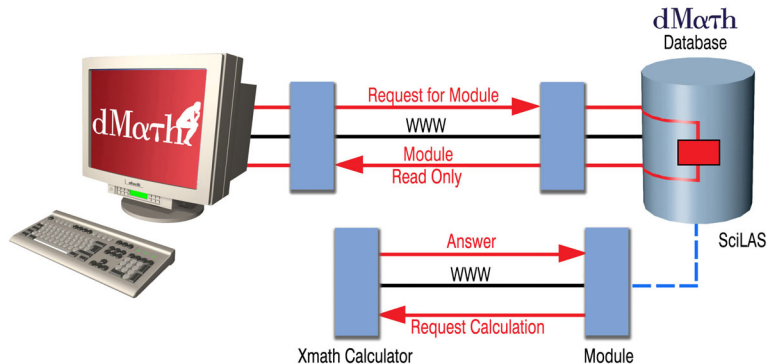


Figure 1. Information server component of the dMathArchive.

■ Xmath Step-by-Step Calculator

The Xmath calculator uses *Mathematica* packages loaded into *webMathematica*. The page design and menus are contained in XML files. Expressions are broken down into different levels using the *Mathematica* object *TreeForm*. All levels are identified in the output by Level n . On each level different mathematical rules (e.g., the substitution rule) may apply and the rule is identified by the package. Users can determine the number of levels in the input. Level 1 indicates the first hint for eventual further calculation by hand.

This is quite similar to what a professor would do on the blackboard; all rules and intermediate calculations are given. This is quite convenient for ODL applications where a professor is not nearby for face-to-face discussion. This has been tested in distance learning in mathematics for school teachers in Norway; a report may be ordered from anne.norstein@hisf.no.

The Xmath calculator contains a variety of mathematical topics, but not all of them give intermediate results. Some topics are still being developed. The algebra topic includes collecting terms, simplifying expressions, expanding expressions, polynomial factorization, partial fractions, step-by-step partial fractions, equation solving, step-by-step equation solving, solved problems, applications, assessments (mathematical comparison of given and answered expressions), case studies, and student projects.

The function topics include plotting, exact solving, numeric solve limits, step-by-step limits, differentiation, step-by-step differentiation, difference quotients, Newton's method, Taylor series, integration, step-by-step integration, definite integrals, Riemann sums, trapezoidal rule, Simpson's rule, solved problems, applications, assessments, case studies, and student projects.

Other topics will be plane curves, functions with several variables, and differential equations. Many of these will have step-by-step facilities.

□ Examples from the Calculator

The calculator can integrate, differentiate, solve equations, and so on for a wide range of expressions. Different levels are shown by scrolling and focusing only on the first level will give the user the first hint in a hand calculation. The program has discovered expressions the calculator can solve for integration but *Mathematica* cannot (certain substitutions).

Level 1

Find the integral $\int \frac{1}{(-3+x)^2} dx$

Substitution Rule, Composite function

Substitute $u = -3 + x$

This gives: $dx = du$, $\int \frac{1}{(-3+x)^2} dx = \int \frac{1}{u^2} du$

Level 2

Find the integral $\int \frac{1}{u^2} du$

Power Rule

$$\int u^n du = \frac{1}{n+1} u^{n+1}, \text{ Here } n = -2$$

$$\int \frac{1}{u^2} du = -\frac{1}{u}$$

Substituting back $u = -3 + x$: $-\frac{1}{u} = -\frac{1}{-3+x}$

Result, Substitution Rule Composite function (Answer)

Substitute $u = -3 + x$

$$\int \frac{1}{(-3+x)^2} dx = \int \frac{1}{u^2} du = -\frac{1}{u} = -\frac{1}{-3+x}$$

Figure 2. web*Mathematica* output for integration.

Here is another example focusing on the first level.

Level 1

Find the integral $\int \frac{x^2}{\sqrt{1-x^6}} dx$

Substitution Rule, Composite function

Substitute $w = x^3$, $dx = \frac{dw}{3x^2}$, $\int \frac{x^2}{\sqrt{1-x^6}} dx = \int \frac{1}{3\sqrt{1-w^2}} dw$

Figure 3. Integration with the first hint for further calculation by hand.

The third example from Xmath uses partial fractions.

Find the Partial Fractions expansion of $\frac{x+2}{x(x^2+4)}$

Task 1

Finding Roots

Finding the roots of the denominator to be used in the denominators of the partial fractions, solving: $x(x^2+4) = 0$

Solution: $x \in \{0, -2i, 2i\}$

Task 2

Partial Fractions

Divide into partial fractions:

$$\frac{x+2}{x(x^2+4)} \equiv \frac{C}{x} + \frac{B+Ax}{x^2+4} \equiv \frac{x(B+Ax) + C(x^2+4)}{x(x^2+4)} \equiv \frac{(A+C)x^2 + Bx + 4C}{x(x^2+4)}$$

This gives: $(A+C)x^2 + Bx + 4C \equiv x + 2$

Task 3

Undetermined Coefficients

Using the method of undetermined coefficients (identical powers of free variable), we get the following system of linear equations:

$$4C = 2$$

$$B = 1$$

$$A + C = 0$$

Solving: $\left\{ B \rightarrow 1, C \rightarrow \frac{1}{2}, A \rightarrow -\frac{1}{2} \right\}$

Answer

$$\frac{x+2}{x(x^2+4)} \equiv \frac{1}{2x} + \frac{1-\frac{x}{2}}{x^2+4}$$

Figure 4. webMathematica output for partial fractions.

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Odd Bringslid is an associate professor at Buskerud University College, Norway. He has also been the leader of the EU projects Xmath (2002–2004) and dMath (2003–2006). From 1981–1983, he was the leader of governmental educational projects, and a third-party developer for Hewlett Packard calculators (1992–1994). He has also received several scholarships including a 1995–1997 post-doctoral scholarship.

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