

BEYOND SUDOKU

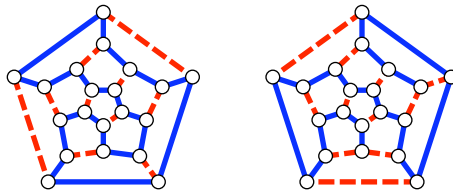
The Icosian Game, Revisited

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Some extensions of Hamiltonian tours are explored.

■ The Original Icosian Game

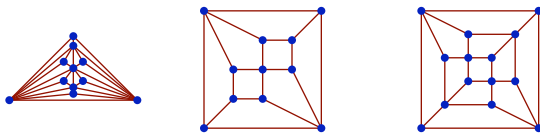
In 1857 Sir William Rowan Hamilton invented the Icosian game [1]. In a world based on the dodecahedral graph, a traveler must visit 20 cities, without revisiting any of them. Today, when the trip makes a loop through all the vertices of the graph, it is called a *Hamiltonian tour* (or *cycle*). When the first and last vertices in a trip are not connected, it is called a *Hamiltonian path* (or *trail*). The first image shown is a tour; the second is a path.



Hamiltonian tour

Hamiltonian path

Hamiltonian cycles gained popularity in 1880, when P. G. Tait made the conjecture: “Every cubic polyhedron has a Hamiltonian cycle through all its vertices”. Cubic means that three edges meet at every vertex. Without the cubic requirement, there are smaller polyhedra that are not Hamiltonian. The simplest counterexample is the rhombic dodecahedron. Every edge connects one of six valence-four vertices to one of eight valence-three vertices. The six valence-four vertices would need to occupy every other vertex in the length-14 tour. Six items cannot fill seven slots, so this is impossible.



Goldner–Harary

Herschel

rhombic dodecahedral

Any noncubic graph can be made cubic by placing a small disk over the exceptions.

