Random Series in Computer Graphics

There may be many ways in which randomness contributes to the beauty of a picture. We show how correlation properties of a random series affect the visual attractiveness of patterns generated by the series and give some examples of our recent designs made with *Mathematica*.

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Using random patterns and placing elements randomly in a picture is an old idea. Computers make it fairly easy to do. However, this kind of design necessarily appears to have been created artificially and hardly resembles human art. Something is missing in such pictures. They are soulless. At present, we do not know in general what to add to make a computer-simulated random pattern mimic one created either by humans or by nature. However, we think we are able to make a suggestion. Briefly, the technique is to add "memory" to the random sequence.

One way to describe mathematically the memory hidden in a sequence of random numbers is by the spectral density or correlation between distant points. In this article, we first introduce a few types of random sequences we will employ and then design a few simple patterns using these sequences in order to explain our approach. We will see how spectral shape of a random sequence affects the degree to which the pattern appears "interesting" and "natural." These qualities are difficult to measure, or even explain, but we will try anyway.

We also present some examples of actual "random" designs made with *Mathematica*. Some of the examples are taken from our collection of textile patterns which has recently been adopted for this year's Spring and Summer women's fashion here in Japan.

Memory in Random Sequences

Three basic types of random processes that are often present in physical systems are known as white ($1/f^0$), flicker ($1/f$), and random walk ($1/f^2$) noises. They are distinguished by the forms of the dependence of the power spectral density $S(f)$ on the frequency f (see Figure 1). In a white sequence, points are absolutely uncorrelated, while in both $1/f$ and $1/f^2$ sequences, points are correlated over *any* time scale. Due to the divergence of the integral $\int_0^\infty S(f)\,df$ at the low or high (or both) frequency limits, none of these types could exist in nature in its "pure" form. However, they do describe real processes in a limited frequency range. White noise got its name from the analogy with white light (in which all spectral components contribute equally to the total power). Sometimes this analogy is extended and flicker noise is referred to as "pink." In general, any noise with "memory" is called "colored noise."

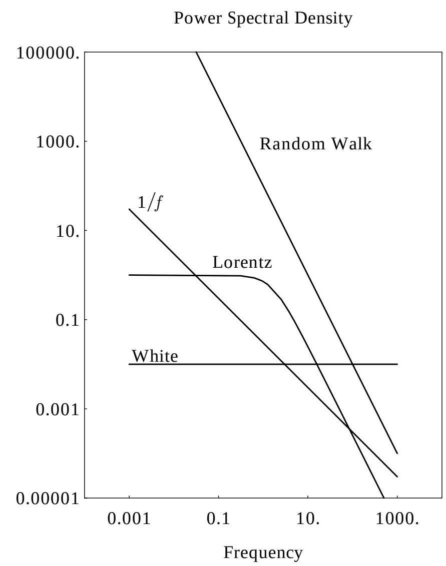

FIGURE 1. Power spectra for white, flicker, random-walk, and Lorentz sequences.

An n-point sequence with a white spectrum can be simulated by a sequence of random numbers:

```
white[n_] := Table[Random[] .5, {n}]
```

Individual points in the sequence are not correlated and, therefore, the spectrum is flat. An n-point $1/f^2$ sequence can be generated by a random walk:

```
RandomWalk[n_] :=
      FoldList[Plus, 0, Table[Random[], {n}] - .5] // Rest
```

This form of accumulation of random increments is essentially an integration of the original (white) sequence, which results in a $1/f^2$ type of spectrum.

Things are more complicated with $1/f$-type sequences. At present, there is no universal explanation of this type of noise, though it is certainly a universal phenomenon. It can be found almost in any physical system, as well as in biological, meteorological, and other systems. Both classical and rock music feature the same $1/f$ law [Voss and Clarke 1975]. Though

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the origin of the $1/f$ spectrum is unknown, sequences possessing this kind of spectrum can easily be simulated with a computer. The recipe is to transform a white noise into the frequency domain with the `Fourier` function, multiply the result point-wise with a Fourier-transformed $1/\sqrt{t}$ kernel, and then take the product back to the time domain with `InverseFourier` (for a good overview of the subject, see the paper by William H. Press [1978]).

Random "Hedgehogs": From Spectrum to Picture

Let us see how memory reveals itself in a picture. In Figure 2, we show random "hedgehogs," which are polar plots of random sequences with a given spectrum. They are displayed with the function `PolarListPlot`, from the standard package ``Graphics`Graphics` ``. The length of all the sequences is chosen to be 100.

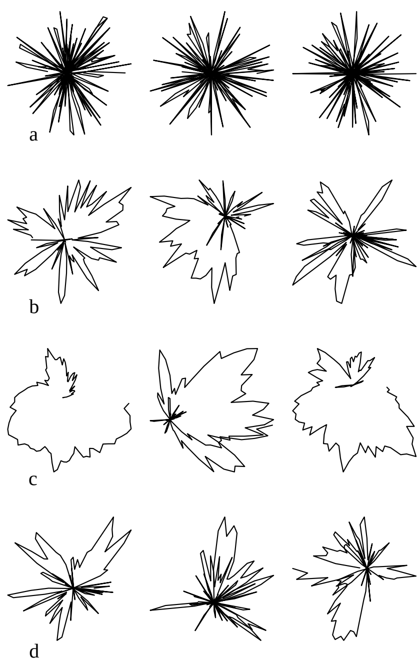

FIGURE 2. White (a), flicker (b), random-walk (c), and Lorentz (d) hedgehogs.

We can see that complete randomness (white noise) produces patterns that are hard to distinguish from each other. Adding correlation (either of $1/f$ or $1/f^2$ type) makes each hedgehog look individual. The difference between the $1/f$ and $1/f^2$ patterns is not prominent, but it certainly exists: the latter are more monotonous and the former are in between the latter and the white noise patterns. We could say that the flicker hedgehogs combine randomness and predictability in the "right" proportions. This intermediate character of $1/f$ noise makes the resulting pattern more stable (perhaps at the expense of "originality").

The $1/f$ type of spectrum is certainly not the only way to balance the "originality" of $1/f^2$ noise with the predictability of white noise. Another possibility comes from an understanding that "predictability" means the convergence of the integral $\int_0^\infty S(f)\,df$ at low frequencies and that "originality" comes from a strong correlation between points ($1/f^2$ behavior) at the high frequency part of the spectrum. We therefore suggest that another suitable candidate for graphics design could be random sequences possessing the Lorentz type of spectrum,

$$S(f) \propto \frac{\tau}{1+(2\pi f\tau)^2},$$

which is white at low frequencies ($f << 1/2\pi\tau$) and has asymptotic $1/f^2$ behavior at high frequencies (Figure 1).

There are a number of ways to generate a Lorentzian sequence. Here, we use a somewhat modified random walk in which random increments are added with a weight:

```
Lorentz[n_, c_] :=
      NestList[# c + Random[](1-c)&, 0, n] - .5 // Rest
```

To get "steady state" in such a system one should drop the first few elements of the list (more for higher values of the weight `c`). Figure 2d shows some Lorentz hedgehogs and confirms that the flicker and Lorentz hedgehogs look quite similar.

Colored Noise in Color Arrangements

We illustrate our approach with one more example. Starting with a regular square lattice of dots, we increment their positions by elements of a random matrix whose rows and columns have the given spectrum. The colors of the dots are decided by a random matrix of the same form. The matrices can be generated by the function:

```
array[fun_, {n_, m_}] :=
      Table[fun[m], {n}] + Transpose[ Table[fun[n], {m}] ]
```

where `fun` is a pure function (such as `White` or `RandomWalk`) that creates a list with the desired spectrum. As can be seen from the flicker and random-walk dots in Figure 3 (next page), correlation in positions and colors seems to make the pattern more "natural" (or, at least, less artificial).

Applied and Pure Art

The following pages show some pieces of our "random designs." It must be emphasized that the most interesting results have been obtained with the $1/f$-type of random sequences. The idea of using this type of sequence in fashion design is due to T. Musha, and the designs are now sold under the trademark "T. Musha Print," a trademark of Yuragi Kenkyusho. The graphic design and *Mathematica* programming are by I. S. Bakshee.

References

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