# Playing with Infinity

**"Great fleas have little fleas upon their back to bite 'em and little fleas have lesser fleas and so ad infinitum."**

**—Augustus De Morgan**

*Philip Chapnick*

*Ad Infinitum: The Ghost in Turing's Machine: Taking God Out of Mathematics and Putting the Body Back In: An Essay In Corporeal Semiotics*, by Brian Rotman. Stanford University Press, Palo Alto, CA, 1993. $39.50. ISBN 0-8047-2127-0; paper: $12.95. ISBN 0-8047-2128-9.

*Zero to Lazy Eight: The Romance of Numbers*, by Alexander Humez, Nicholas Humez, and Joseph Maguire. Simon and Schuster, New York, 1993. $21.00. ISBN 0-671-74282-5.

Comprehending the mysteries of mathematical infinity has challenged the mathematical imagination since at least the 5th century B.C. when Zeno of Elea formulated his celebrated paradoxes. Undoubtedly, even before these formal conundrums furrowed the philosophical brow, people wondered just how far the sky above them extended or how distant it was to the opposite shores of the vast oceans into which the sun rose and set with unerring regularity. Speculation about the infinite, especially in its guise as the unending, the incompleteable, is a universal human experience. It is unique to mathematics, though, for the infinite to play such a concrete and thoroughly practical role.

After a 2,000-year hiatus from mainstream mathematical thinking, the infinite took center stage in mathematics with the invention of the calculus in the 17th century and then again several hundred years later with the development of modern set theory and analysis in the late 19th and early 20th centuries. Cantor's breathtaking leap into the transfinite showed us how to count the infinite; his sublimely simple proof that the real line is uncountable remains one of the most profound yet accessible proofs in all of mathematics.

In Cantor's view, "The first, simplest fact, accessible to everyone, on which the theory of the transfinite is based, is the simultaneous boundlessness and yet the definiteness in itself of the series of all finite cardinal numbers 1, 2, 3, ...,$v$, ... viewed as a constant set of clearly differentiated things." Brian Rotman's book, while unfortunately not as accessible as Cantor's proof, attacks this basic premise with force and vigor from a strikingly post-modern perspective.

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Rotman ingeniously argues that the physical limits to counting to infinity as a realizable computation define a region beyond which indeterminateness reigns. There are numbers, and processes derived from numbers, that are too large to be operated upon in our physical universe and are thus beyond the scope of certain (apodictic, Kant would say) knowledge. Using finite—but often rather long, complex, and clausally convoluted—sentences, Brian Rotman develops a most ingenious refutation of the naturalness of the natural numbers in his latest work: *Ad Infinitum: The Ghost in Turing's Machine: Taking God Out of Mathematics and Putting the Body Back in: An Essay in Corporeal Semiotics.* This essay is well worth the parsing effort. Brian Rotman is one of the most creative contemporary philosophers of mathematics thinking today.

Briefly put, he argues that the uniform infinite extent of the natural numbers is not natural at all. Resonating with two of the most important themes in current scientific thinking, nonlinear dynamics and computational complexity, Rotman develops a new strongly finitistic—and somewhat startling—view of the nature of the mathematical infinite. He insists that the physical processes of entropy and the corresponding physical limits of computation must be taken into account in explaining how mathematicians prove and communicate theorems. He fashions his sophisticated semiotic approach to mathematical discourse in order to bypass key deficiencies in each of the three dominant philosophical orthodoxies in interpreting mathematics: Platonism, formalism and intuitionism. A sophisticated triadic model of mathematical discourse lets him reinterpret each of these schools as having too simple-minded an acceptance of the infinite.

Rotman is sympathetic to the basic thrust of Wittgenstein's way of philosophizing about mathematics—as a specialized language game for making assertions about mathematical inscriptions. He focusses his analysis on one particular move in the game: the use of the ellipsis (...) to represent the unending extension of a the sequence of integers off to infinity, the "ad infinitum." Though sympathetic to Wittgenstein, Rotman's approach is far richer and more substantial in its detail. He reinterprets mathematical proof as a type of conscious waking dream, a controlled thought experiment. The approach via semiotics—the study of how symbols embody meaning—is most directly descended from the seminal work

of the 19th century American philosopher Charles Sanders Peirce, but Rotman's formulation is highly original and provocative.

For Rotman, mathematics is a written language, but essentially different from ordinary written language whose function is to record and describe actual speech. Mathematical symbols do not record thoughts; they embody them. Taking an explicitly deconstructionist attitude, Rotman asserts that mathematical symbols are essentially self-referential (like music), yet he avoids the sterility of the formalist approach by situating mathematical discourse in a social/historical matrix. By viewing mathematics as a form of inscriptional activity inescapably embedded in a social framework, it's utility becomes less surprising; it arises naturally from the evolution of its symbolism over thousands of years of use and development.

A brief summary such as this one can only skim the surface of this wonderfully wide-ranging work. Reaching far beyond the analysis of the infinite—certainly of major importance in itself—is the utility of the underlying framework that Rotman creates to understand the nature of mathematical discourse. I hope Rotman and others take up where he has left off and develop his semiotic approach to address the full richness and scope of the contemporary mathematical enterprise.

If this book appeals to you, I'd also recommend highly Rotman's earlier book, *Signifying Nothing: The Semiotics of Zero*, thoughtfully reprinted this year by Stanford University Press to accompany his latest work. In this essay, Rotman shows how the dual role of Zero, as both an ordinary integer and a significant point of origin, accounts for its unique and sometimes paradoxical properties.

And now for something totally different: *Zero to Lazy Eight: The Romance of Numbers*. This book is terrific to take on an airplane trip. It's a mathematical confection for the mind, but in the spirit of the 1990s, it's a "health food" candy so you can feel good about devouring it. I did. This idiosyncratic book is obviously the product of rambling fascinating intellect. It is essentially a series of essays, ostensibly about the numbers 1 through 13 (and a brief unsatisfying chapter about infinity). Each essay free-associates about number words and idiomatic expressions, rambling amiably through linguistics, folklore, mathematics, religion, social habits, and so on.

Though they seem like (barely) controlled streams of consciousness, the essays will keep you fascinated as they meander from the sayings of Gorge Carlin and Woody Allen, through Thomas Aquinas and Lao Tzu, all in the context of numbers and the way in which they have insinuated themselves into the fabric of our ordinary language.

A reviewer of a previous book by the extraordinary Humez brothers, characterized them as "manic digressives" but don't let that deter you. This fabulous chamber of curiosities will delight and inform you about cartography (the four-color problem), timekeeping (take five), the history of billiards (behind the eight ball), the evolution of fashion (the whole nine yards), and much more. And if you're interested in etymology, you won't find a more thoroughly enjoyable and easy-to-read guidebook to the origins of number words and associated phrases.