Seismic tomographic studies have determined three-dimensional (3D) velocity structures, in detail, of the crust and the upper mantle of the Earth. Yet, simple two-dimensional (2D) sections have generally been used to present 3D tomographic results. Here we show 3D views and animations of the Earth’s structure that are made as easy as 2D sections by using Mathematica. As an example, low-velocity zones in the upper mantle are shown in three dimensions together with major volcanoes, mid-crustal reflectors, earthquake hypocenters, the Moho discontinuity, and the upper plane of the subducted slab that are observed in northeastern Japan. Low velocities in volcanic areas generally correspond to high temperature and indicate possible presence of magma. The 3D animations enable us to investigate the special correlation between
low-velocity zones, volcanoes, reflectors, earthquakes, and the slab, and thus enable us to study magma ascent pathways in detail.

Introduction

Earthquakes generate seismic waves of compressional and shear modes (P and S waves, respectively). Much of the knowledge about the internal structure of the Earth comes from earthquake observations, as seismic waves propagate throughout the globe. Many seismic stations on the surface of the Earth together yield a large amount of seismic data, which enable observers to determine accurate locations of earthquake hypocenters and accurate travel time of seismic waves. Seismologists thus determine velocities of seismic waves propagating through the Earth. Generally, waves travel faster through cold material than they do through hot. Temperature heterogeneity exists in the Earth because cold lithosphere (plate) subducts at the trench and high-temperature lava erupts at volcanoes. Therefore, high- or low-seismic velocity regions are present within the Earth, and differences from the average velocity (called velocity perturbations) are determined from seismic studies. An image of velocity perturbations of the inner Earth is called seismic tomography. Analogous to X-rays in medical tomography (CT: computerized tomography), seismic rays are used to probe the Earth in seismic tomography. Differences in velocity allow scientists to calculate the size, density, and elastic properties of the Earth’s interior and use that information to make predictions about the shape, materials, and thermal conditions of the Earth. The knowledge of the internal structure is essential to understand geologic processes like plate tectonics and continental drift.

Recently, major advances in the study of seismic tomography have been made with the advent of computer software and hardware. One of them is the development of three-dimensional (3D) seismic tomography, which unveils many important aspects of the Earth. In general, however, many 3D tomographic results have been shown in two-dimensional (2D) sections. 3D presentations are only rarely made. In 2D sections, it is not easy to investigate, for example, the spatial correlation between low- (or high-) velocity regions and important geophysical processes (for example, earthquake hypocenter, volcano location, and plate movement). Such correlation, however, is clearly seen and studied in detail in a 3D view.
In this paper, we present a simple, practical, and effective way to show a 3D view and animation of the Earth’s structure by using Mathematica. We select the region of northeastern (NE) Japan where interesting features typical to a volcanic area are observed. They are the presence of mid-crustal S wave reflectors and the occurrence of low-frequency microearthquakes \cite{1, 2}. The reflectors are thin magma bodies located at about 8 to 15 km beneath the surface. Due to the acoustic impedance difference between magma and surrounding crustal rocks like granite, S waves are distinctly reflected on the magma-rock interface. Microearthquakes of magnitude less than 2.5 are frequently observed in and around the low-velocity zones, and have relatively low-characteristic frequencies (around 2 Hz), compared with the ordinary seismic frequency band (up to ~20 Hz). These observations indicate the close relationship between magnetism, slow-velocity anomalies, low-frequency microearthquakes and S wave reflectors \cite{2, 3}. Here, the low-velocity zones are shown in three dimensions, and are discussed in relation to the occurrence of low-frequency microearthquakes and the presence of mid-crustal S wave reflectors. The 3D views suggest magma ascent pathways in the Japan island arc.

### 3D Structure and Animation

As the cold Pacific plate subducts beneath NE Japan, high-temperature magma is produced and causes an eruption of an island arc volcano. Many observed seismic activities, either large- or microearthquakes, have led seismologists to determine the detailed velocity structure beneath Japan \cite{2, 4}. For example, Zhao and others \cite{4} analyzed 14,045 arrival times including both P and S waves. Complex seismic velocity discontinuities (the Moho discontinuity and the upper plane of the subducting Pacific plate) are taken into account in their 3D velocity inversion. Their results have shown low-velocity zones in the crust beneath the volcanoes and in the central part of the mantle wedge, and up to 6 percent higher velocity in the Pacific plate. Their tomography results, given in FORTRAN code, yield a value of velocity perturbation in percentages at any point of the upper mantle beneath NE Japan. For a 3D animation, here we use their 3D P wave velocity structure.

Figures 1 (a) to (e) show a 3D mapping of the Earth’s interior to 120 km depth beneath NE Japan (lat. 36.5-41.6 N, long. 138.5-142.5 E) based on the P wave
seismic tomography by Zhao and others [4]. In this figure, the vertical scale is exaggerated 3.2 times so that the details of the internal structure can be seen clearly. Programs are given in Listings 1 to 3 for 3D views and animations. Even simple Mathematica code yields 3D Earth structure in considerable detail. At the top of the figures (the Earth’s surface), the location of active volcanoes (red circles) and the map of NE Japan are shown. To be transparent, the Moho discontinuity and the surface of –3% velocity perturbation are described, not by planes, but by green and pink points, respectively. The Moho discontinuity is a boundary between the crust and the mantle. The upper plane of the subducted Pacific plate is shown by a combination of ten light blue rectangles. In Listings 1 to 3, dv-3 is a set of points of –3% velocity perturbation, while dv-4 and dv-5 consist of the data tracing outlines of the north-south cross sections of –4% and –5% velocity perturbations. These low-velocity regions are shown in polygons (yellow and orange, respectively) with contour lines that reduce the data quite a lot (compared with points) and help to save memory. Within the yellow and orange polygons, velocity perturbations are less than –4% and –5%, respectively. The contour lines (light purple, the data file dv-4c) trace outlines of –4% velocity perturbation at every 1 km depth. These low-velocity data can be shown in dots, too; however, the memory needed for 3D views and animations becomes quite large.

(a)
active volcanoes
large crustal earthquakes (M$\geq$6)
low-frequency microearthquakes
mid-crustal S wave reflectors
Moho discontinuity

Figure 1. Three-dimensional mapping of low-velocity zones and seismic activities in northeastern Japan as viewed from (a) above and (b) below from southwest direction, from (c) above and (d) below from northwest direction, and from (e) below from west (symbols are described in (e)). Vertical scale is exaggerated 3.2 times. It is observed that fairly low-velocity zones exist locally beneath the sites of volcanoes and downwards to the west. The high temperature regions (regions of less than $-3\%$ velocity perturbation) appear to extend to greater depths in the mantle wedge, suggesting a plume-like uprising of hot mantle rocks. In between the volcanoes and the low-velocity zones, S wave reflectors exist and low-frequency microearthquakes occur. Two major low-velocity zones are observed below 60 km depth and branch off at the uppermost mantle, and eventually underplate the Moho right beneath the volcanoes.

Listing 1. Programs for displaying 3D structure (Figure 1) of the Earth’s interior beneath northeastern Japan. Parameter values given in ViewPoint give Figure 1a. The files, dv-3, dv-4, dv-5, dv-4c, moho, slab, microearthq, largeearthq, reflector, volcano, and NEJapan consist of the data of $-3\%$ velocity perturbation, less than $-4\%$ velocity perturbation, less than $-5\%$ velocity perturbation, $-4\%$ velocity perturbation, the Moho discontinuity, the upper plane of the subducting Pacific plate (slab), hypocenters of
microearthquakes, hypocenters of large crustal earthquakes, S wave reflectors, locations of active volcanoes, and the coastline of northeastern Japan, respectively.

dv1={PointSize[0.001],RGBColor[1,0.3,0.7],Map[Point,<<dv-3]}
dv2={EdgeForm[],FaceForm[RGBColor[1,1,0],RGBColor[1,1,0]],
      Map[Polygon,<<dv-4]}
dv3={EdgeForm[],FaceForm[RGBColor[1,0.74,0],
      RGBColor[1,0.74,0]],Map[Polygon,<<dv-5]}
dvc={Thickness[0.0008],RGBColor[0.6,0.4,0.5],
       Map[Line,<<dv-4c]}
moh={PointSize[0.0065],RGBColor[0.3,0.9,0],
      Map[Point,<<moho]}
slb={EdgeForm[],FaceForm[RGBColor[0.8,1,1],
      RGBColor[0.8,1,1]],Map[Polygon,<<slab]}
meq={PointSize[0.01],RGBColor[0.2,0,0.77],
      Map[Point,<<microearthq]}
leq={PointSize[0.014],RGBColor[0.3,0.3,0.3],
      Map[Point,<<largeearthq]}
ref={FaceForm[RGBColor[0.9,0.63,1],RGBColor[0.9,0.63,1]],
      Map[Polygon,<<reflector]}
vol={PointSize[0.02],RGBColor[1,0,0],Map[Point,<<volcano]}
nej={Thickness[0.0065],Map[Line,<<NEJapan]}
a=0.15; b= -12; c=170
For[i=0,i<c,   {If[i<17.5,{a=0.3,b= -15}],
      If[And[17.5<i,i<32.5],{a=0.2,b= -13.3}],
      Show[Graphics3D[{dv1,dv2,dv3,dvc,moh,slb,meq,
          leq,ref,vol,nej}],
        PlotRange→ {{138.5,142.5},(36.5,41.6),(-120,0)},
        BoxRatios→ {1,1.5,1.1},
        ViewPoint→ {-3.6,i*a+b,3.2},
        Lighting→ False};i++]

▲ Listing 2. Programs for displaying 3D movie (Movie 1) of the Earth's interior beneath northeastern Japan from SSW to NNW.

dv1={PointSize[0.001],RGBColor[1,0.3,0.7],Map[Point,<<dv-3]}
dv2={EdgeForm[],FaceForm[RGBColor[1,1,0],RGBColor[1,1,0]],
      Map[Polygon,<<dv-4]}
dv3={EdgeForm[],FaceForm[RGBColor[1,0.74,0],
      RGBColor[1,0.74,0]],Map[Polygon,<<dv-5]}
dvc={Thickness[0.0008],RGBColor[0.6,0.4,0.5],
       Map[Line,<<dv-4c]}
moh={PointSize[0.0065],RGBColor[0.3,0.9,0],
      Map[Point,<<moho]}
slb={EdgeForm[],FaceForm[RGBColor[0.8,1,1],
      RGBColor[0.8,1,1]],Map[Polygon,<<slab]}
meq={PointSize[0.01],RGBColor[0.2,0,0.77],
      Map[Point,<<microearthq]}
leq={PointSize[0.014],RGBColor[0.3,0.3,0.3],
      Map[Point,<<largeearthq]}
ref={FaceForm[RGBColor[0.9,0.63,1],RGBColor[0.9,0.63,1]],
      Map[Polygon,<<reflector]}
vol={PointSize[0.02],RGBColor[1,0,0],Map[Point,<<volcano]}
nej={Thickness[0.0065],Map[Line,<<NEJapan]}
c=157
For[i=0,i<c,   {If[i<17.5,{a=0.3,b= -15}],
      If[And[17.5<i,i<32.5],{a=0.2,b= -13.3}],
      Show[Graphics3D[{dv1,dv2,dv3,dvc,moh,slb,meq,
          leq,ref,vol,nej}],
        PlotRange→ {{138.5,142.5},(36.5,41.6),(-120,0)},
        BoxRatios→ {1,1.5,1.1},
        ViewPoint→ {-3.6,i*a+b,3.2},
        Lighting→ False};i++]}
If[And[32.5<i,i<124.5],{a=0.15,b= -11.7}],
If[And[124.5<i,i<139.5],{a=0.2,b= -17.9}],
If[139.5<i,{a=0.3,b= -31.8}],
Show[Graphics3D[{dv1,dv2,dv3,dvc,moh,slb,meq,leq,ref,
vol,nej}],
PlotRange → {{138.5,142.5},{36.5,41.6},{-120,0}},
BoxRatios → {1,1.5,1.1},
ViewPoint → {-3,0.15,i*a+b},
Lighting → False];i++]

▲ Listing 3. Programs for displaying 3D movie (Movie 2) of the Earth’s interior beneath northeastern Japan, from the sky to the inside of the Earth from the west.

Here we focus our attention on low-velocity zones (where temperatures are potentially high, as described earlier), because they are closely tied to magma source regions and volcanoes. In Figure 1, S wave reflectors (purple rectangles) hypocenters of large crustal earthquakes (brown circles) and low-frequency microearthquakes (black points) are also shown. Although the actual size and shape of the reflectors are smaller and more irregular than those shown in the figures, they are represented by dipping rectangular planes. Large crustal earthquakes of magnitude above 6 (M > 6) are based on observations since 1931 [3], and have caused at least some damage in property and human life.

For 3D animations, we successively change the parameters of the function
ViewPoint as described in Listings 2 and 3. The speed, range, and axis of the rotation of the 3D structure are varied by the values, a, b, and c. In Movie 1 (Movie1.nb), y values in ViewPoint are varied from –12 to 13.35 at every 0.15 (see Listing 2), and there are 170 figures viewing from SSW to NNW. On the other hand, z values in ViewPoint are varied from –15 to 15 at every 0.15, 0.2, and 0.3 in the ranges of |z| < 6.9, 6.9 < |z| < 9.9, and 9.9 < |z| < 15, respectively (see Listing 3), in Movie 2 (Movie2.nb). This yields 157 figures viewed from the sky to the inside of the Earth from the west. Increasing a value with increasing absolute value of z yields a relatively homogeneous rotation of the 3D structure. For clear presentation of Movies 1 and 2 on the screen or on the monitor, we have used larger parameter values in RGBColor in Listings 2 and 3 than in Listing 1, and microearthquakes and large earthquakes are shown in blue and gray circles, respectively. The movies enable us to see the structure from various directions and to investigate the spatial relations between volcanoes, earthquakes, reflectors, and low-velocity zones in detail.
Distribution of Volcanoes and Hot Low-Velocity Zones, and Mantle Dynamics

Some interesting features observed in the 3D structure beneath NE Japan are summarized as follows. At 110 km depth, fairly low-velocity regions (velocity perturbations below –4%) appear in the mantle wedge and exist to shallow depths right beneath the Moho discontinuity. Extremely low-velocity regions (velocity perturbations below –5%) are observed around 70 km depth beneath lat. 40.2 N, long. 140 E, and around 40 km depth right beneath the volcanoes. These low-velocity regions appear to be localized. Low velocity indicates high temperature (and so low density) and the possible presence of magma in the region. Diapir-like melting regions of up to a few tens of kilometers are expected along the middle of the mantle wedge. The most extensive melting may occur in the uppermost mantle (~40 km depth) beneath the volcanoes. The continuous supply of magma from melting zones may cause the surface volcanic activity of the island arc. However, most of the magma will cool at depth and form the island arc crust. The local existence of hot low-velocity zones observed in this study may account for the lifetime of the island arc volcanoes. Once a melting zone cools down, volcanic activity will cease. New volcanic activity may start again after another diapir in the mantle wedge rises up to the Moho discontinuity.

The contours of –3% velocity perturbation are also shown in Figure 1 and Movies 1 and 2. Within the contours, velocity perturbations are below –3%, and thus higher temperatures and lower densities than average are expected. These high-temperature regions indicate the upwelling of hot mantle materials from beneath. Two major upwelling regions exist below 60 km depth at 38N and 40N, branch off at the uppermost mantle, and eventually underplate the Moho right beneath the volcanoes. The results indicate that melting zones localized in the upwelling hot mantle materials cause volcanic activity in the NE Japan arc.

In between the volcanoes and the low-velocity zones, mid-crustal S wave reflectors exist, and low-frequency microearthquakes occur in the crust, as also shown in Figure 1. The close relationship of magma source regions with S wave reflectors and microearthquakes indicates that magma ascent from the uppermost mantle to the ductile lower crust may cause low-frequency microearthquakes, and magma penetration into the brittle upper crust may produce the mid-crustal
reflectors, as has been discussed previously [2]. Magma movement in the brittle upper crust may increase stress (or at least change the state of stress) in the crust, and may contribute to the occurrence of crustal earthquakes including destructive earthquakes. Interestingly, large crustal earthquakes \( (M > 6) \) also occur around the volcanoes and the low-velocity zones (Figure 1, Movies 1 and 2).

## Summary

The 3D movies are a powerful tool to investigate the internal structure of the Earth. *Mathematica* offers a simple and convenient way to show 3D animations for all sorts of tomographic data. The 3D animations are easily shown on portable computers; therefore, it is possible to present them in lectures, meetings, exhibitions, schools, fieldwork, and so on.

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## References


## Additional Material

Available at www.mathematica-journal.com.

- **Movie1.nb.** Movie of the 3D Earth’s structure beneath northeastern Japan from SSW to NNW. Vertical scale is exaggerated 3.2 times. When running the movie, select the animation direction of the forward-backward mode (see The Mathematica Book: Section 2.10.12). The animation should run from the first cell to the last, and should then reverse back to the first cell again.

- **Movie2.nb.** Movie of the 3D Earth’s structure beneath northeastern Japan from the sky to the inside of the Earth from the west. Vertical scale is exaggerated 3.2 times. When running the movie, select the animation direction of the forward-backward mode.